HW 2 Help

28. ORGANIZE AND PLAN Given the hypotenuse and the angles we will use the trigonometric function $\sin\theta = \frac{opposite}{hypotenuse}$ to solve for each of the unknown sides. We note that the 60° is the angle opposite the long side S_L because it is a wider angle and 30° is the angle opposite the short side S_S.
We are interested in the unknown sides so we will use a little algebra to find that *opposite* = *hypotenuse* × sin θ. We should also recall the sin of the common angles used in this problem (table 3.1). sin 30° = 1/2 and $\sin 60° = \sqrt{3}/2$.
SOLVE Applying our trigonometric relation for right triangles we find:

$$S_L = 12.5 \text{ cm} \times \sin 60° = 12.5 \text{ cm}\frac{\sqrt{3}}{2} = 10.8 \text{ cm}$$

and

$$S_S = 12.5 \text{ cm} \times \sin 30° = 12.5 \text{ cm}\frac{1}{2} = 6.25 \text{ cm}$$

REFLECT We can check the answer by using the Pythagorean Theorem: $S_L^2 + S_S^2 = (12.5 \text{ cm})^2$.

32. ORGANIZE AND PLAN We need to find the right triangle lurking within the problem. From where you are the peak of the mountain is θ above the horizon. The map puts you a horizontal distance *x* from the peak. So you have an angle and a side adjacent to the angle. We need to find the side opposite the angle to find the height, *y* above the present elevation. The elevation will then be the elevation of the car at the point of measurement, plus the length of the side opposite the angle.
The trigonometric function that relates the two things we know to the side we don't is the tangent function:
$\tan\theta = \frac{y}{x}$. Isolating for the unknown value yields:

$$y = x \tan\theta$$

SOLVE The height above the current elevation is:

$$y = x \tan\theta = 25.0 \text{ km} \times \text{tab } 3.1° = 1.35 \text{ km}$$

The peak's elevation is then 1350 m + 1580 m = 2930 m
REFLECT This elevation is a respectably sized peak, though one of the smaller ones in the Rocky Mountains.

36. **ORGANIZE AND PLAN** We are given a displacement vector in cardinal directions, the time it takes to traverse the displacement and asked to find the displacement and the average velocity. The first part is just converting the cardinal directions to the Cartesian form. We recall that North, South, East and West correspond to the unit vectors $\hat{\imath}, -\hat{\imath}, \ \hat{\jmath}$ and $-\hat{\jmath},$ respectively. The average velocity (which we remember is a vector) will be determined by dividing the displacement vector by the time of traversal. We are asked to find the average velocity in units of m/s so we will also need to convert minutes to seconds.

SOLVE Mapping the cardinal directions to the Cartesian unit vectors yields displacement *r* in Cartesian form:

$$\vec{r} = 1250 \text{ m}\hat{\imath} - 900 \text{ m}\hat{\jmath}$$

Converting 20 min → *t* s we have $\frac{60 \text{ s}}{1 \text{ min}} = 1$ so

$$20 \cancel{\text{min}} \times \frac{60 \text{ s}}{\cancel{\text{min}}} = 1200 \text{ s}$$

The average velocity is then:

$$\frac{\vec{r}}{\Delta T} = \frac{1}{1200 \text{ s}}(150 \text{ m}\hat{\imath} - 900 \text{ m}\hat{\jmath}) = 1.04 \text{ m/s}\hat{\imath} - 0.75 \text{ m/s}\hat{\jmath}$$

or, in cardinal directions 1.04 m/s east and 0.75 m/s south.

REFLECT This problem required two types of conversions: minutes to seconds and cardinal directions to Cartesian form. Acquiring skill in unit conversions will be something you thank yourself for the rest of your life.

39. **ORGANIZE AND PLAN** Subtracting vectors is achieved by “combining like terms” where the unit vectors are treated as variables. The difference between vectors $\vec{v}_1$ and $\vec{v}_2$ is a vector given by $\vec{v}_1 - \vec{v}_2 = (v_{1x} - v_{2x})\hat{\imath} + (v_{1y} - v_{2y})\hat{\jmath}$

SOLVE

$$\vec{r}_1 - \vec{r}_2 = (r_{1x} - r_{2x})\hat{\imath} + (r_{1y} - r_{2y})\hat{\jmath} = (2.39 + 3.56) \text{ m}\hat{\imath} + (-5.07 - 0.98) \text{ m}\hat{\jmath} = 5.95 \text{ m}\hat{\imath} - 6.05 \text{ m}\hat{\jmath} = \vec{r}_1 - \vec{r}_2$$

REFLECT Sign is important when subtracting components and sometimes can be tricky. You can always check the answer by drawing the vectors tail to tail and obtaining $\vec{r}_1 - \vec{r}_2$ by drawing a vector starting at $\vec{r}_2$ and ending at $\vec{r}_1$. Unlike addition, $\vec{r}_1 - \vec{r}_2 \neq \vec{r}_2 - \vec{r}_1$ so order matters.

48. **ORGANIZE AND PLAN** There is a distinction between distance traveled and displacement. Distance traveled is the reading on the odometer of the car (assuming it started at zero at the beginning of the race). Displacement is the displacement vector beginning at the start and ending at the location of interest.

The distance travelled will just be the arc of a circle which is given by fractions of a circumference. The circumference of a circle is $C = 2\pi R$. One-quarter lap is corresponds to $\frac{C}{4}$, one-half lap $\frac{C}{2}$ and one whole lap C.

The displacement is a vector and requires both magnitude and direction. The displacement vector $\vec{d}$ can be obtained by calculating the difference between the final position vector and the initial position vector, $\vec{d} = \vec{r}_f - \vec{r}_i$.

Once we have the position vectors at the start, at one-quarter lap, one-half lab and one full lap it is a straight forward matter to calculate the vector difference.

SOLVE Distance traveled:

quarter lap: $\frac{C}{4} = \frac{\pi R}{2} = 393\ \text{m}$

half lap: $\frac{C}{2} = \pi R = 785\ \text{m}$

full lap: $C = 2\pi R = 1570\ \text{m}$

Position vectors in component form (r_x, r_y) at:
start: (250 m, 0)
quarter lap: (0, 250 m)
half lap: (–250 m, 0)
full lap: (250 m, 0)

Displacement vectors from start to
quarter lap: $(0, 250\ \text{m}) - (250\ \text{m}, 0) = (-250\ \text{m}, 250\ \text{m}) = -250\ \text{m}\hat{i} + 250\ \text{m}\hat{j}$

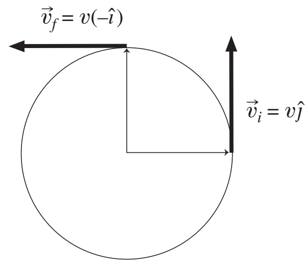
half lap: $(-250\ \text{m}, 0) - (250\ \text{m}, 0) = (-500\ \text{m}, 0) = 500\ \text{m}\hat{i}$

full lap: $(250\ \text{m}, 0) - (250\ \text{m}, 0) = (-0, 0) = \vec{0}$

REFLECT This problem reinforces the difference between distance traveled (a scalar quantity) and displacement. The stark difference is seen between the distance traveled to go all the way around, one circumference; and the displacement associated with ending upwhere you started, zero.

58. ORGANIZE AND PLAN The average acceleration over an duration of time requires the difference between the velocities at the start and end of the duration as well as the duration itself.

The velocities are obtained from the speed and direction at the beginning of the race and 1/4 of the way around the circle. If the race starts with cars facing due north (which will call the $\hat{j}$ direction) then 1/4 of the way around the circle the car will be facing due west (which we will call the $-\hat{i}$ direction) (see figure below).



Race track

The time is determined by using rate × time = distance. We can use the geometry of the problem to determine the total distance traveled and we are given the rate of travel over this distance.

The acceleration is then just obtained by dividing the change in velocity by the time.

We also note that the units are km/h. To obtain an answer that we can compare to experience we want to convert everything to meters and seconds.

SOLVE The difference in velocities is:

$$\Delta\vec{v} = 90\ \text{km/h}(-\hat{i} - \hat{j}) = \frac{90\ \text{km/h} \times 1000\ m \times 1\ \text{hr}}{1\ km \times 3600\ s}(-\hat{i} - \hat{j}) = 25\ \text{m/s}(-\hat{i} - \hat{j})$$

The path length d of this part of the race is 1/4 the circumference of a circle:

$$d = \frac{1}{4}2\pi R = \frac{\pi 1.25\ \text{km}}{2} = 1.96\ \text{km}.$$

The time of travel is $\Delta t = \frac{d}{v} = \frac{1.96\ \text{km}}{90\ \text{km/hr}} = 0.0218\ hr = 78\ s$

The average acceleration is:

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{25\ \text{m/s}(-\hat{i} - \hat{j})}{78\ s} = 0.31\ \text{m/s}^2(-\hat{i} - \hat{j})$$

REFLECT The average acceleration is directed toward the south-west direction which jibes with the picture and the trajectory of the car around the track. The magnitude of the acceleration is 0.45 ms^2 or about 5% the acceleration due to gravity on the Earth.

63. **ORGANIZE AND PLAN** Given the initial velocity we can use the relationships derived in question 11 to find the range and the time of flight. The range x_R is given by

$$x_R = \frac{2v_{0y}v_{0x}}{g}.$$

The time of flight was derived to be $T = \frac{2v_{0y}}{g}$.

The maximum height can be determined by considering that half the time time of flight is spent going up and half is spent going down when the launch elevation is the same as the landing elevation. When the projectile is at the apex of the path the component of the velocity in the vertical direction is zero. Using equation 3.19a with zero initial velocity in the direction of acceleration ($y = -\frac{1}{2}gt^2$) and taking the origin of the coordinate system as the apex. The distance of the fall in $t = \frac{T}{2}$ will give us the maximum height.

Known: $\vec{v}_i = 27\ \text{m/s}(\cos 45°\hat{i} + \sin 45°\hat{j})$

SOLVE The range is:

$$x_R = \frac{2v_{0y}v_{0x}}{g} = \frac{2\times(27)^2\,\text{m}^2/\text{s}^2}{2\times 9.8\ \text{m/s}^2} = 74\ \text{m}$$

The time of flight is:

$$T = \frac{2v_{0y}}{g} = \frac{2\times(27)\ \text{m/s}}{\sqrt{2}9.8\ \text{m/s}^2} = 3.9\ \text{s}$$

The maximum height is:

$$y = \frac{1}{2}gt^2 = \frac{1}{2}9.8\ \text{m/s}^2(\frac{3.9\ \text{s}}{2})^2 = 19\ \text{m}$$

REFLECT The distance between home-plate and the outfield wall is approximately 400 ft. The range found above is consistent with a fly-ball hit into the outfield. The time of flight is also consistent with the experience of watching a ball game and timing the time from bat to glove of a fly-ball.

66. **ORGANIZE AND PLAN** Given the initial velocity we can use the relationships derived in question 11 to find the range and the time of flight. The range x_R is given by

$$x_R = \frac{2v_{0y}v_{0x}}{g} = \frac{2v^2 \sin\theta\cos\theta}{g}$$

where θ is the launch angle from the horizontal and v is the speed.

We are asked to determine if an initial velocity and launch angle will result in a range that is less than 25 m. We shall extract v_{0y} and v_{0x} from the given information: $v_i = (20 \text{ m/s}, 15°)$ to obtain whether or not the ball will be in or out of the court.

Given a range and a launch angle, determination of the speed is just algebraic manipulation of the relationship above:

$$v = \sqrt{\frac{gx_R}{2\sin\theta\cos\theta}}$$

Preliminary calculations: $v_{0y} = 20 \text{ m/s} \sin 15° = 5.18 \text{ m/s}$ and $v_{0x} = 20 \text{ m/s} \cos 15° = 19.3 \text{ m/s}$

SOLVE Proof that the ball will land in play: The range of the struck tennis ball is

$$x_R = \frac{2 \times 5.18 \text{ m/s} \times 19.3 \text{ m/s}}{9.8 \text{ m/s}^2} = 20 \text{ m} < 25 \text{ m}$$

The maximum velocity that will result in a ball in play is:

$$v_{\text{max}} = \sqrt{\frac{9.8 \text{ m/s}^2 \times 25 \text{ m}}{2\sin 15° \cos 15°}} = 22 \text{ m/s}$$

REFLECT Another application of the range formula. It is interesting to note that 2 m/s difference in the velocity of the ball coming off the racket makes the different between in and out of the court. Great tennis players must have done very well in their physics classes.

70. ORGANIZE AND PLAN We have developed the range equation given the horizontal and vertical components of the velocity:

$$x_R = \frac{2v_{0y}v_{0x}}{g}$$

This problem is stated in a way that allows a straightforward application of this equations.

SOLVE The range of the long jumper is:

$$x_R = \frac{2 \times 3.85 \text{ m/s} \times 7.5 \text{ m/s}}{9.8 \text{ m/s}^2} = 5.89 \text{ m}$$

REFLECT The worlds record for women's long jump in 1928 was 5.98 meters. Today is 7.52 meters. A distance of 5.89 m is a respectable and reasonable distance.

82. ORGANIZE AND PLAN The equation for the centripetal acceleration is $a_r = \frac{v^2}{R}$. We are given a limit on the allowed value for $a_r < 1.0 \text{ m/s}^2$ leading to the inequality:

$$a_c^{max} > \frac{v^2}{R}$$

Solving for *R*:

$$R > \frac{v^2}{a_c^{max}}$$

$$R_{\min} = \frac{v^2}{a_c^{max}}$$

Notes on units: We will standardize the units in the problem to meters and seconds.

$$100 \text{ km/hr} \times 1000 \frac{m}{km} \times \frac{1\ hr}{3600\ s} = 27.8 \text{ m/s}$$

SOLVE Minimum radius of curvature is $R_{\min} = \frac{v^2}{a_c^{min}} = \frac{(27.8 \text{ m/s}^2)}{1 \text{ m/s}^2} = 773 \text{ m}$

REFLECT A larger radius of curvature can accommodate faster speeds or lower centripetal acceleration.